

Measuring the Distributions of Public Inflation Perceptions and Expectations in the UK

Bayesian Analysis of a Normal Mixture Model
for Interval Data with an Indifference Limen

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Plan

- 1 Motivation and Contributions
- 2 Data
- 3 Model Specification
- 4 Bayesian Analysis
- 5 Results Using 6 or 8 Intervals
- 6 Results Using 18 Intervals
- 7 Use of the Estimated Distributions
- 8 Summary

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Why Inflation Expectations?

Important in monetary economics

- Level
- Disagreement

Why disagree?

- sticky information?
- bounded rationality?

Since Mankiw, Reis, and Wolfers (2004), there are many empirical works on the **distributions** of inflation expectations. They typically use **numerical data** (e.g., Michigan Survey), but many surveys collect only **categorical data**.

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Quantification of Qualitative Data

Ordinal data on inflation expectations (y_i^*)

$$y_i := \begin{cases} 1 & \text{if } y_i^* < 0 \\ 2 & \text{if } y_i^* \approx 0 \text{ (indifference limen)} \\ 3 & \text{if } y_i^* > 0 \end{cases}$$

Quantification of Qualitative Data

Estimate the distribution of $\{y_i^*\}$ using $\{y_i\}$

Need strong assumptions; e.g., Carlson and Parkin (1975)

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Use of Interval Data

Murasawa (2013, OBES)

- 1 Estimate the distributions of inflation expectations among households in **Japan**
- 2 Fit various **unimodal** distributions (normal, skew normal, skew exponential power, and skew t)
- 3 Apply **ML method**

This paper

- 1 Estimate the distributions of inflation perceptions and expectations among individuals in the **UK**
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- 1 Bayesian analysis of a normal mixture model for interval data with an indifference limen (useful for measuring inflation expectations)
Prior settings: hierarchical prior
Posterior simulation: NUTS (No U-Turn Sampler)
- 2 Estimate the distributions of public inflation perceptions and expectations in the UK during 2001Q1–2015Q4
- 3 Illustrate a possible use of the estimated distributions by measuring information rigidity in the UK

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BoE/GfK NOP Inflation Attitudes Survey

- Quarterly survey started in 2001Q1
- Quota sample of adults throughout the UK
- Quota is 4,000 (Q1) or 2,000 (others)
- Use 60 samples (2001Q1–2015Q4) separately

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Intervals

8 intervals (excluding 'No idea'):

- 1 Gone/Go down
- 2 Not changed/change (indifference limen)
- 3 Up by 1% or less
- 4 Up by 1% but less than 2%
- 5 Up by 2% but less than 3%
- 6 Up by 3% but less than 4%
- 7 Up by 4% but less than 5%
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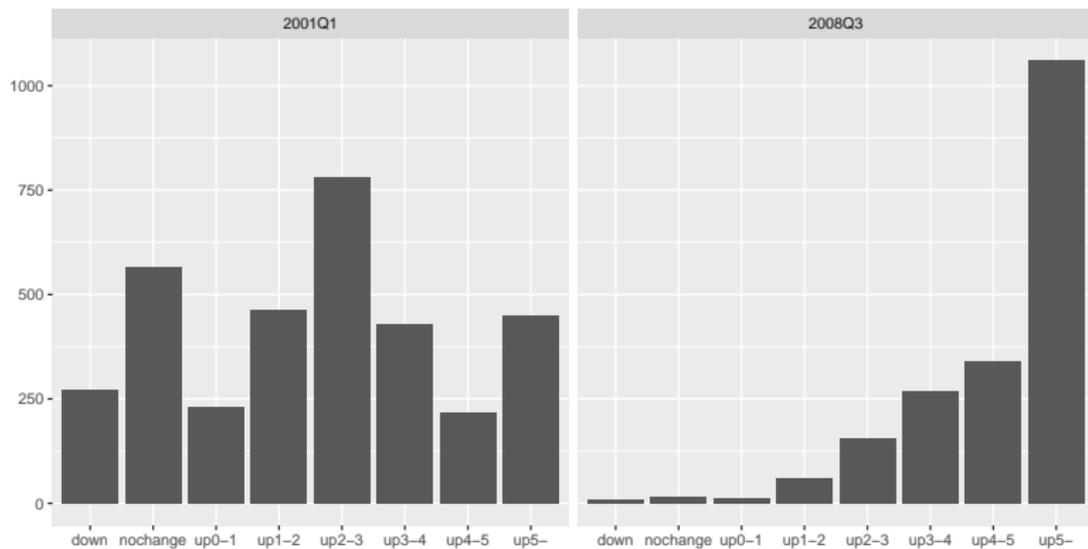
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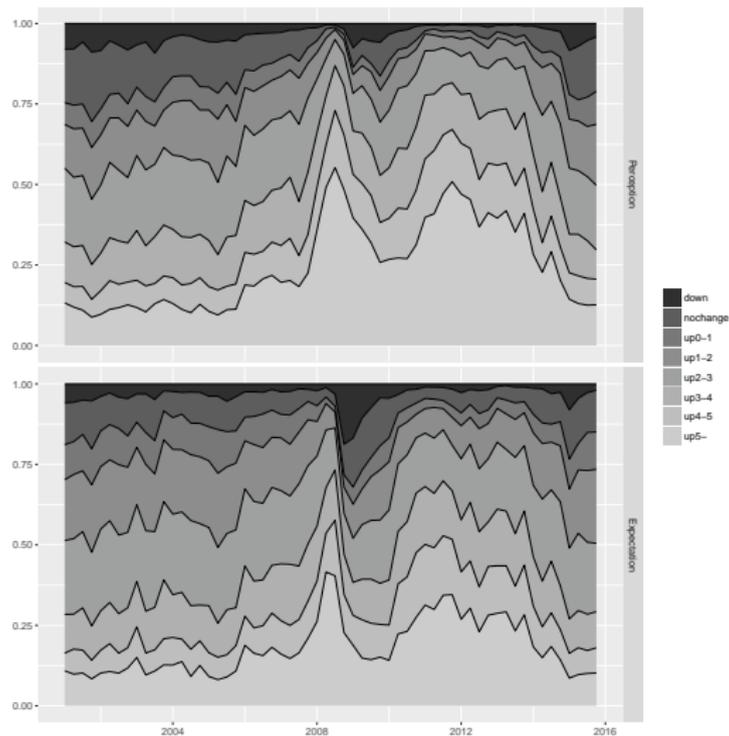
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Inflation Perceptions (2001Q1 and 2008Q3)



Relative Frequencies (2001Q1–2015Q4)



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Interval/Ordinal Response Model

Interval/ordinal response model with an indifference limen

$$y_i := \begin{cases} 1 & \text{if } \gamma_0 < y_i^* \leq \gamma_1 \\ \vdots & \\ J & \text{if } \gamma_{J-1} < y_i^* \leq \gamma_J \end{cases}$$

where

- y_i^* is a latent variable
- $-\infty = \gamma_0 < \dots < \gamma_l < 0 < \gamma_u < \dots < \gamma_J = \infty$
- we know $\{\gamma_j\}$ except for an indifference limen $[\gamma_l, \gamma_u]$

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Normal Mixture Model

Normal mixture model for y_i^*

$$y_i^* \sim \sum_{k=1}^K \pi_k \mathcal{N}(\mu_k, \sigma_k^2)$$

Parameters

- $\boldsymbol{\pi} := (\pi_1, \dots, \pi_K)'$
- $\boldsymbol{\mu} := (\mu_1, \dots, \mu_K)'$
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Why Bayesian?

ML may not work for normal mixture models

- 1 Given numerical data, the likelihood function is **unbounded** with a degenerate component; cf. Kiefer and Wolfowitz (1956)
- 2 Given interval data, the likelihood function is bounded, but the convergence problem remains; cf. Biernacki (2007)

Bayesian analysis seems safer.

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Prior

Weakly informative priors (i.e., proper but vague) for the distribution parameters

$$\pi \sim \text{Dirichlet}(\mathbf{1}_K)$$

$$\mu_k \sim \text{N}(2.5, 100), \quad k = 1, \dots, K$$

$$\sigma_k^2 \sim \text{Inv-Gam}(2, \beta_0), \quad k = 1, \dots, K$$

A hierarchical prior for β_0 (Richardson and Green (1997))

$$\beta_0 \sim \text{Gam}(.2, .1)$$

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$$\gamma_l \sim \text{U}(-\infty, 0)$$

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- 1) Problem with MCMC for **cutoff points** with large samples
 - Gibbs** Extremely slow
 - MH** Faster (with collapsing and reparametrization), but low acceptance rate (20–50%)
 - NUTS** Much faster with high acceptance rate (80–99%), easy to implement using **Stan**
- 2) Label switching problem
 - Component labels may switch during MCMC
⇒ Mixture parameters may not converge
 - Focus on **permutation invariant parameters** (moments and quantiles); cf. Geweke (2007)

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- 2 Estimates of higher-order moments are imprecise.
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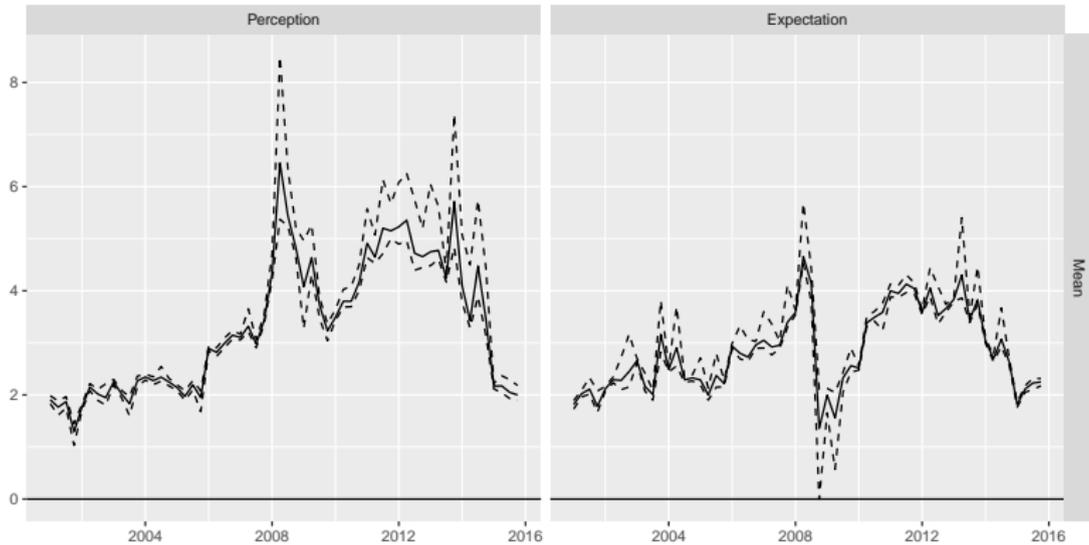
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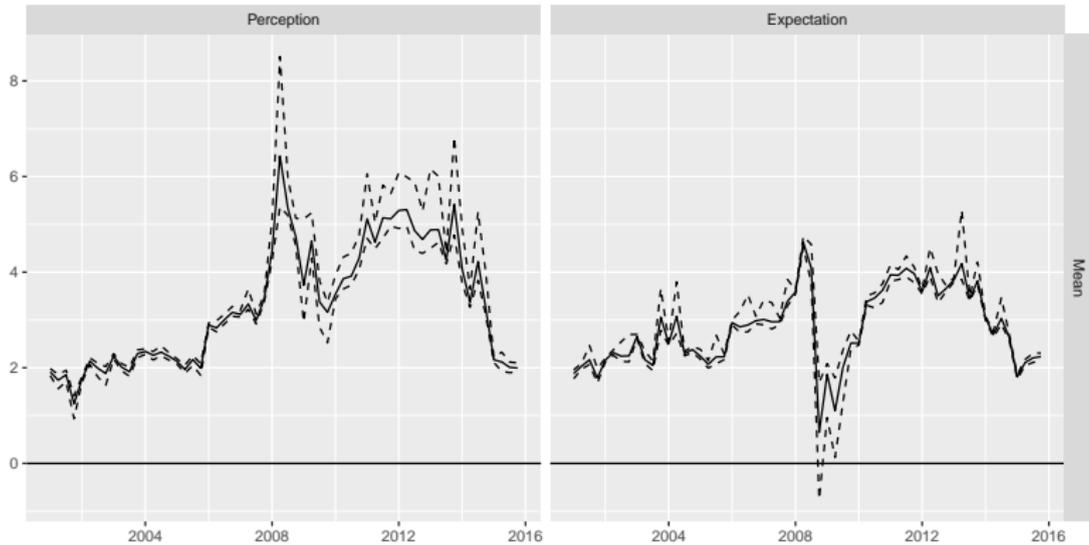
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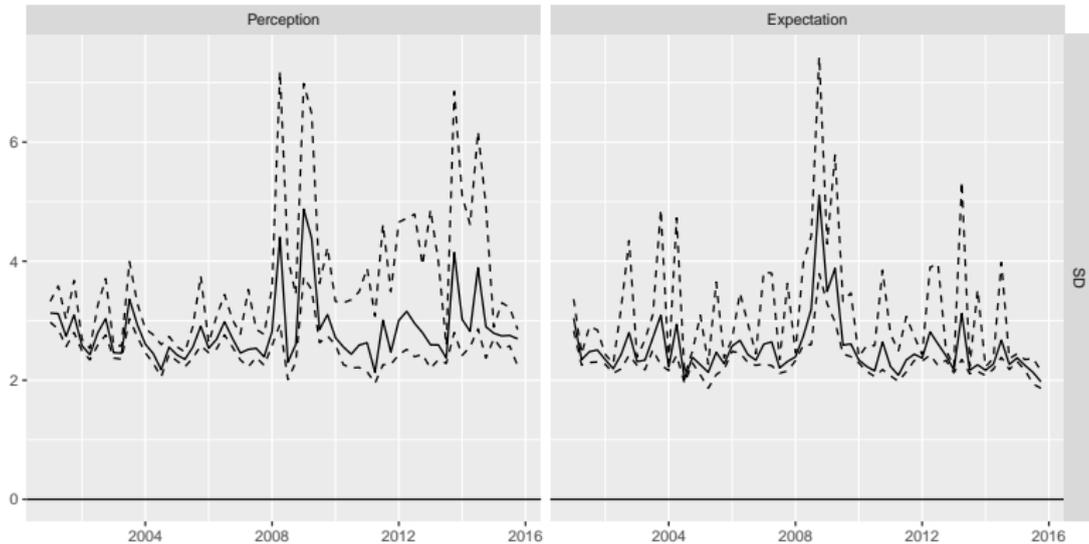
Posterior median and 68% error band

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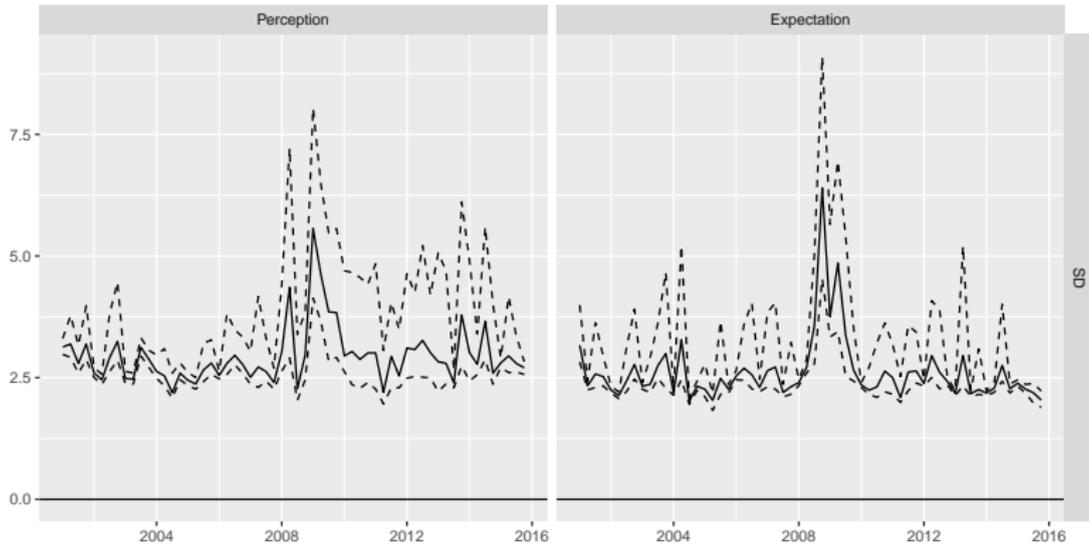
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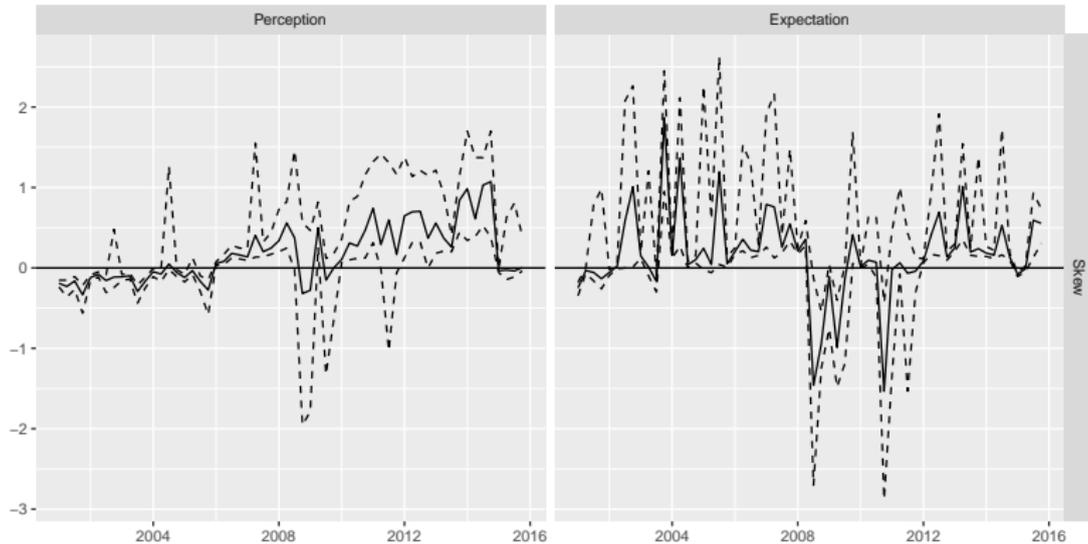
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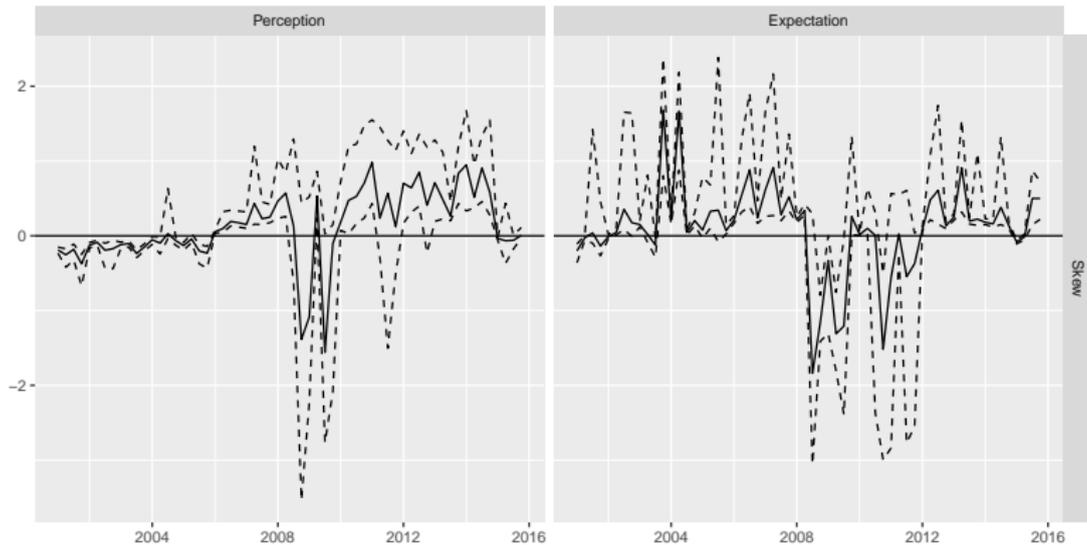
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Skewness (6 Intervals)



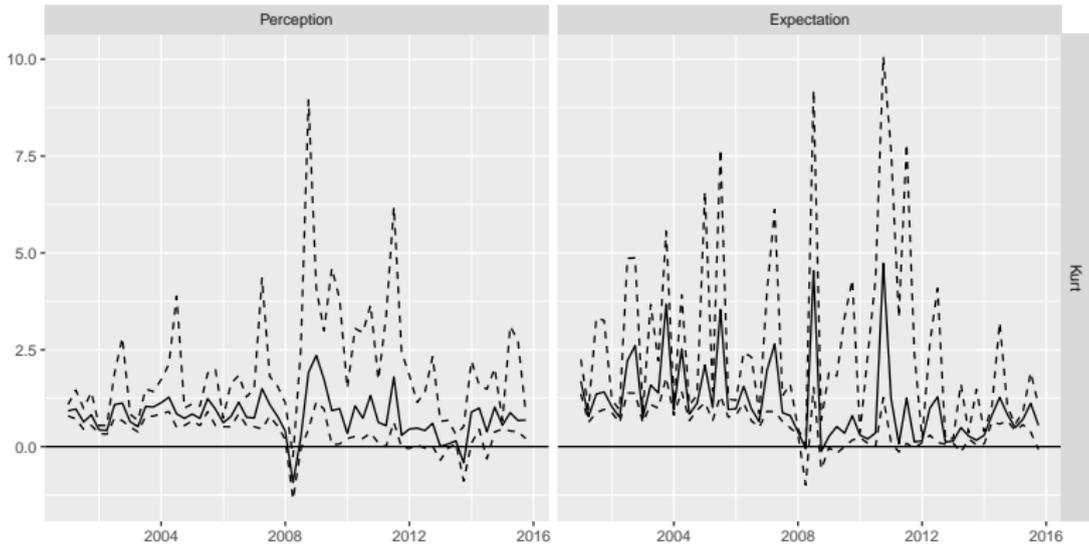
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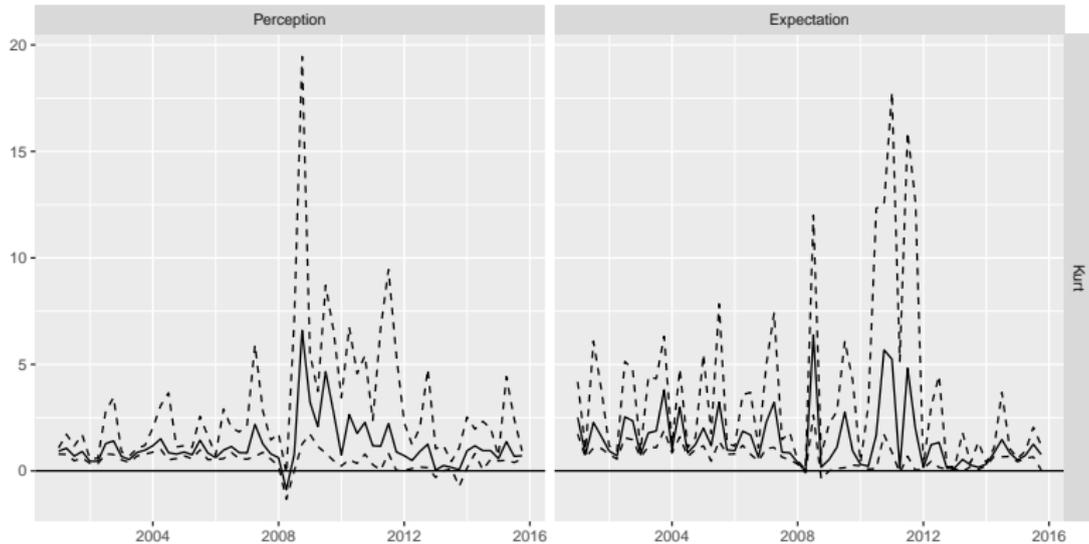
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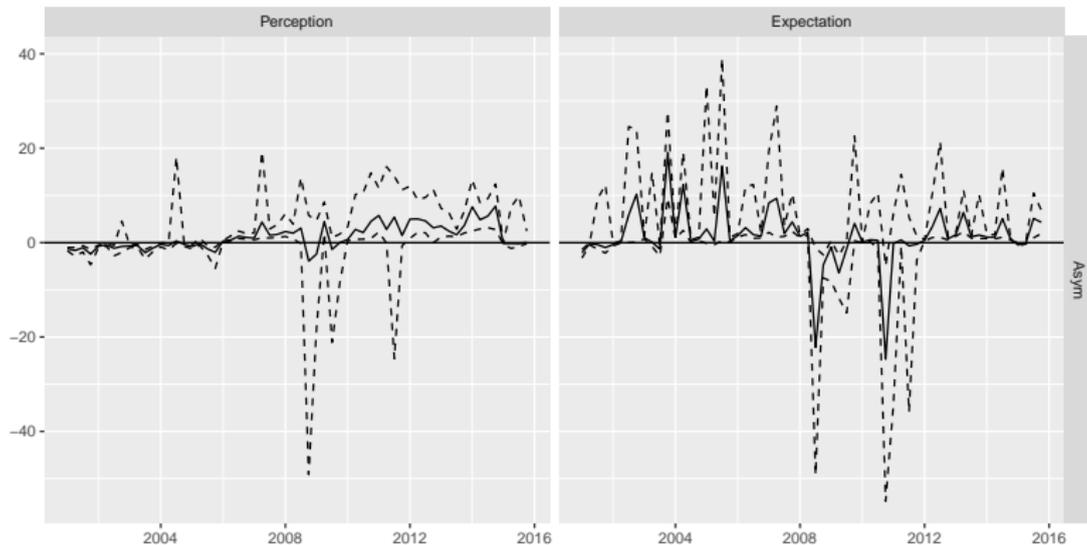
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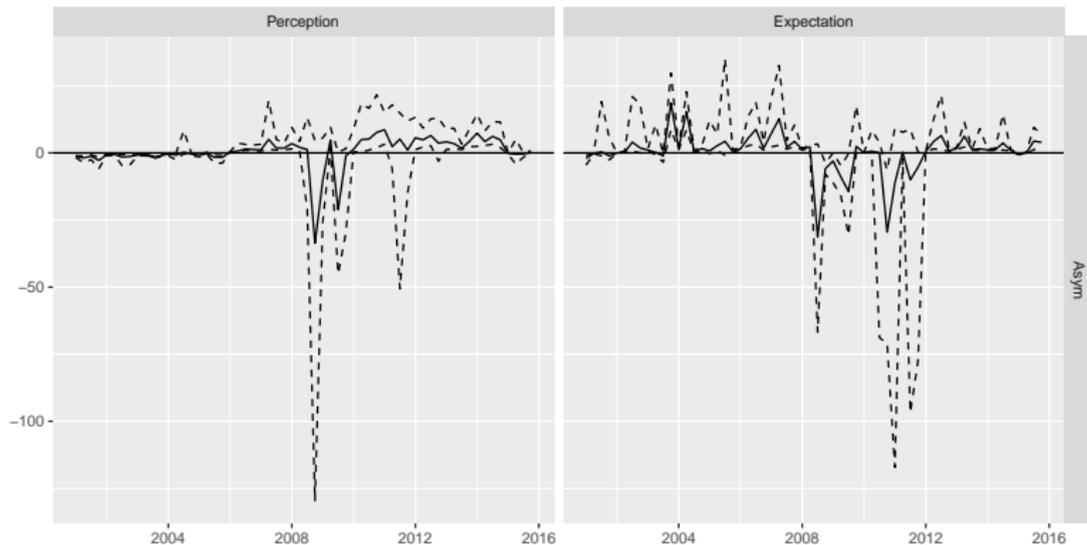
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Asymmetry in the Tails (6 Intervals)



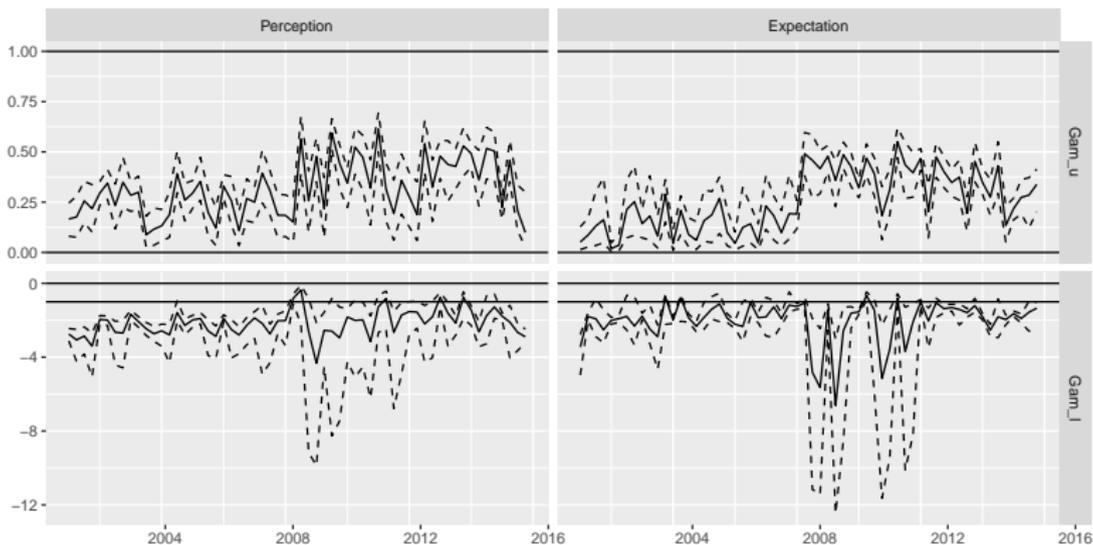
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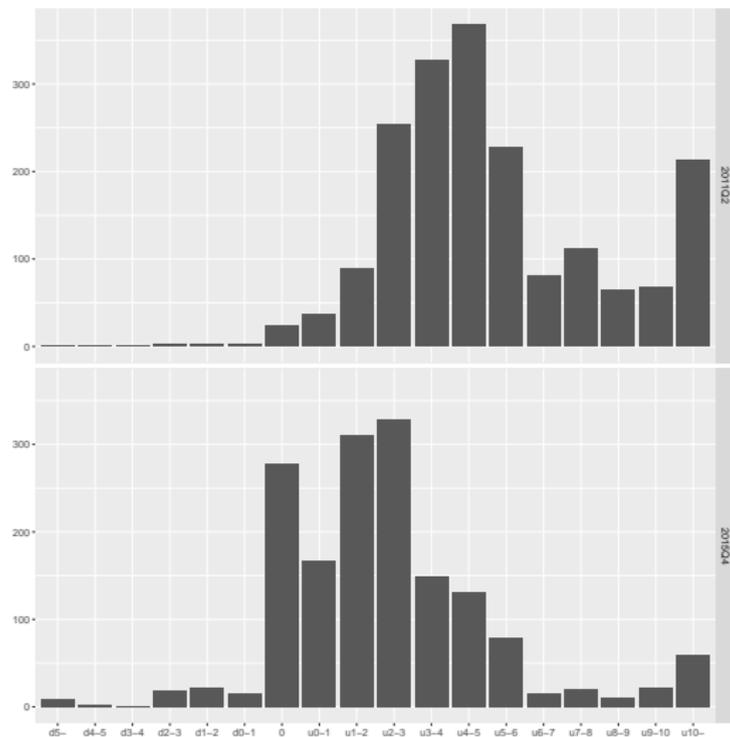


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Inflation Perceptions (2011Q2 and 2015Q4)



Results Using 18 Intervals

- Estimation period: 2011Q2–2015Q4
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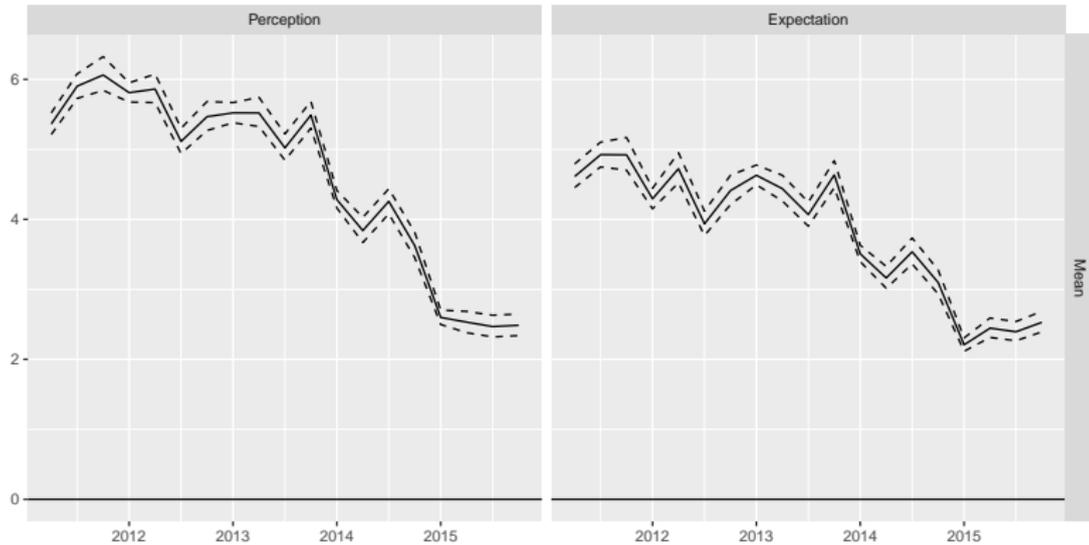
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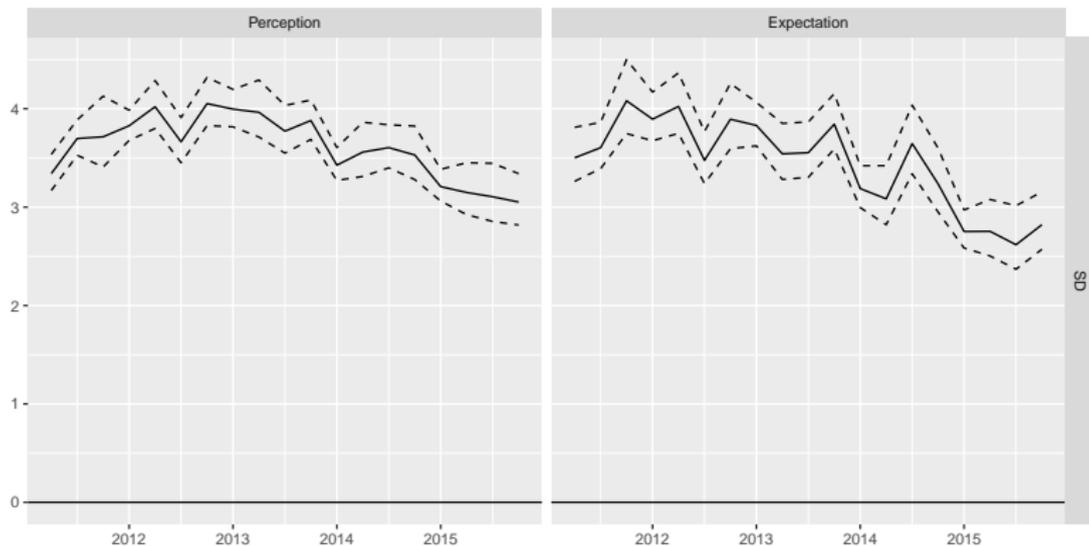
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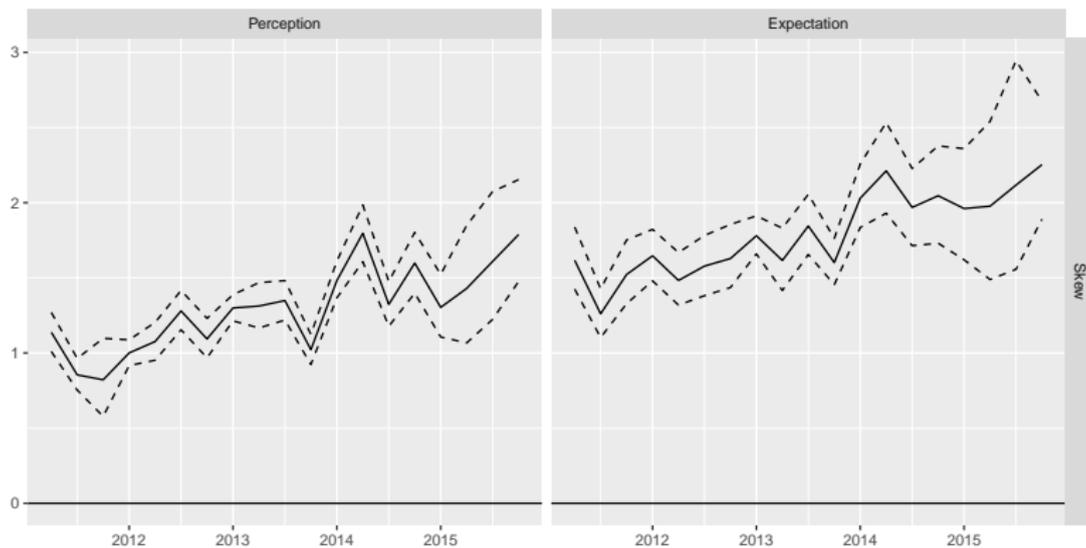
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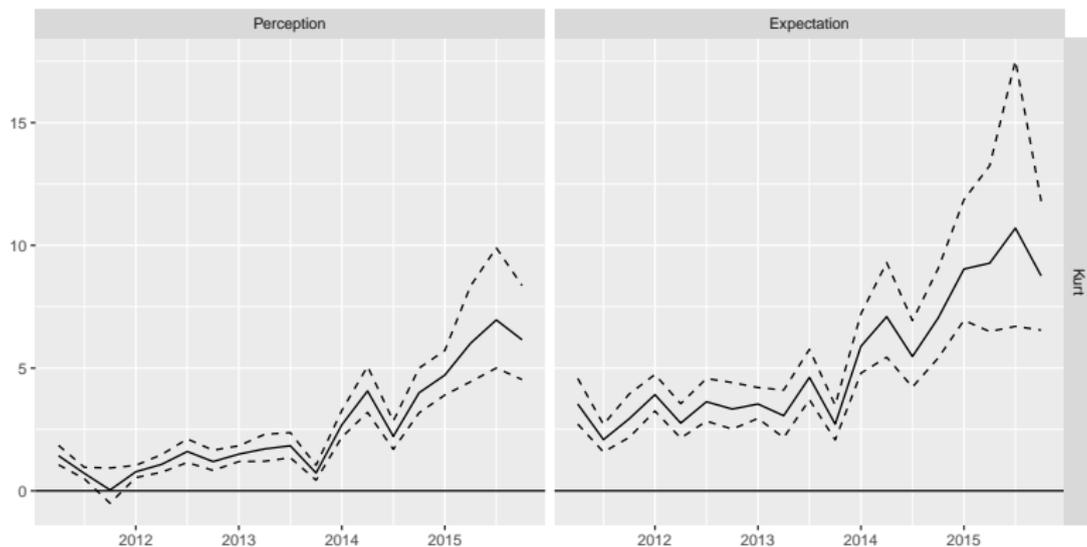
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Skewness



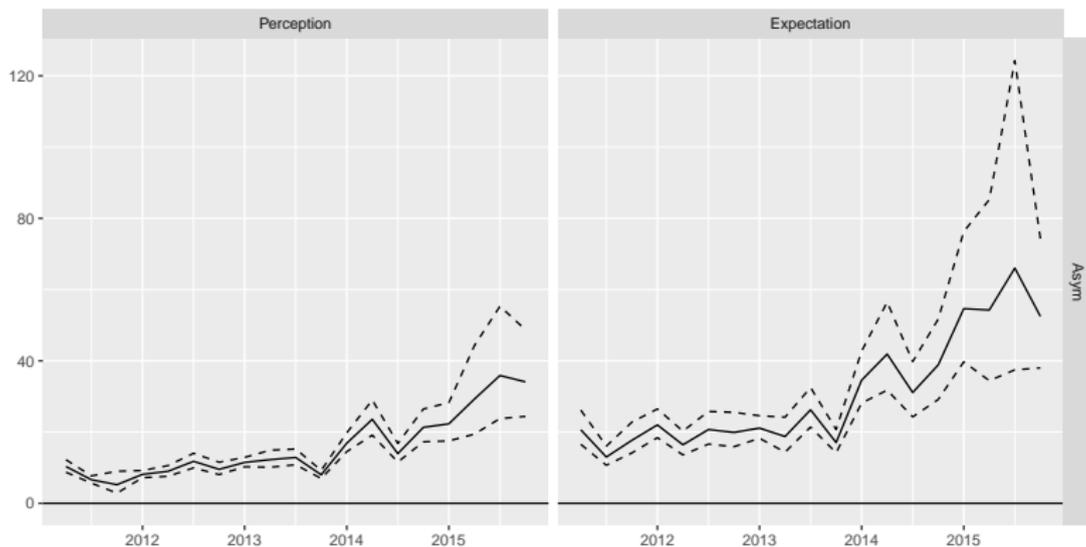
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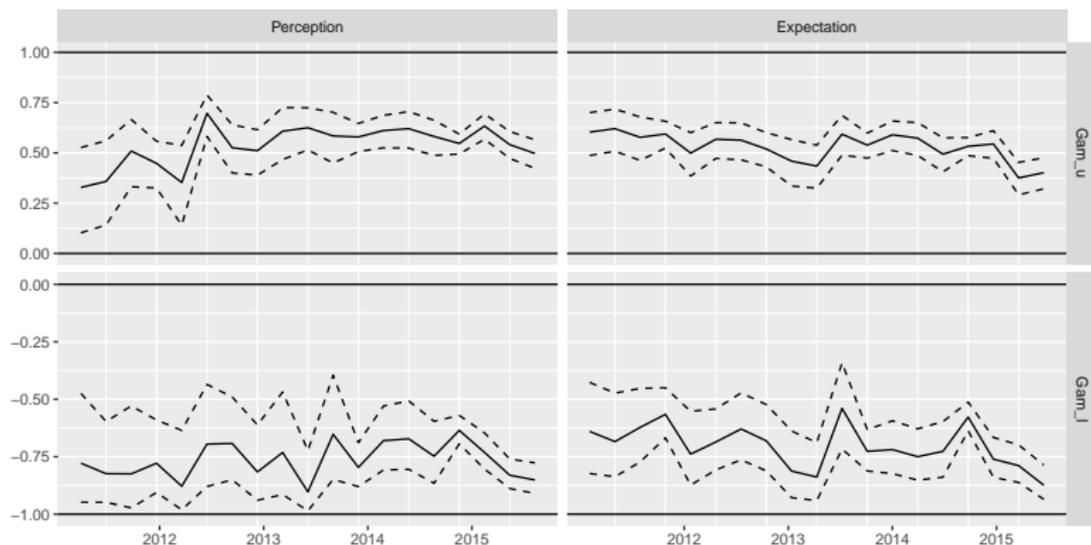
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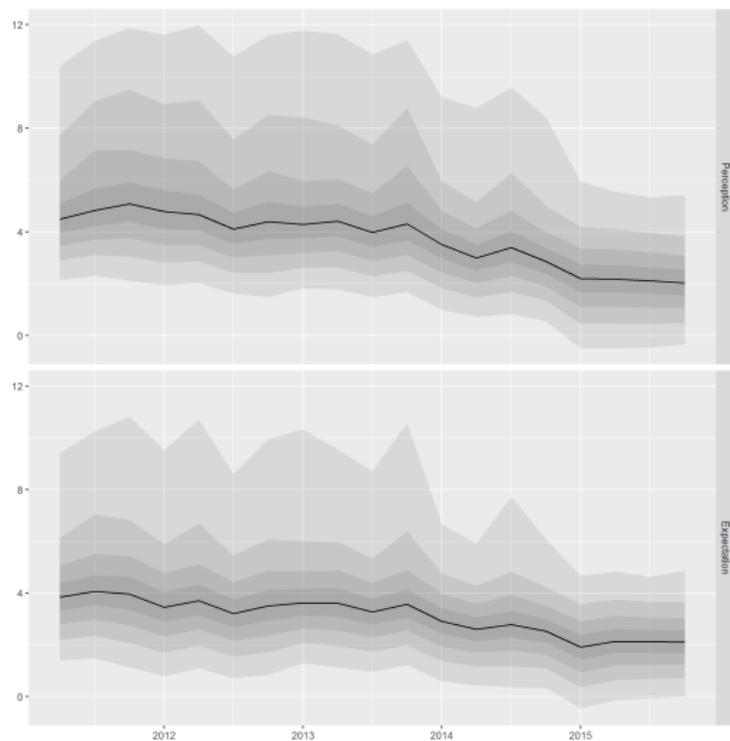
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Indifference Limen



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Can use the mean inflation expectations for further analyses

- testing for **rationality of expectations**
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The results depend on how we specify the distribution of inflation expectations \implies **Use a flexible distribution**
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Use of individual data (now available!)

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