Measuring the Distributions of Public Inflation Perceptions and Expectations in the UK

Bayesian Analysis of a Normal Mixture Model for Interval Data with an Indifference Limen

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Plan

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Why Inflation Expectatoins?

Important in monetary economics

- Level
- Disagreement

Why disagree?

- sticky information?
- bounded ratoinality?

Since Mankiw, Reis, and Wolfers (2004), there are many empirical works on the distributions of inflation expectations. They typically use numerical data (e.g., Michigan Survey), but many surveys collect only categorical data.

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Quantificatoin of Qualitative Data

Ordinal data on inflation expectations (y_i^*)

$$y_i := \begin{cases} 1 & \text{if } y_i^* < 0 \\ 2 & \text{if } y_i^* \approx 0 \text{ (indifference limen)} \\ 3 & \text{if } y_i^* > 0 \end{cases}$$

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Estimate the distribution of $\{y_i^*\}$ using $\{y_i\}$

Need strong assumptions; e.g., Carlson and Parkin (1975)
$\Longrightarrow$ Use interval data instead.

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Murasawa (2013, OBES)

1. Estimate the distributions of inflation expectations among households in Japan
2. Fit various unimodal distributions (normal, skew normal, skew exponential power, and skew t)
3. Apply ML method

This paper

1. Estimate the distributions of inflation perceptions and expectations among individuals in the UK
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Contributions

1. Bayesian analysis of a normal mixture model for interval data with an indifference limen (useful for measuring inflation expectations)
   Prior settings: hierarchical prior
   Posterior simulation: NUTS (No U-Turn Sampler)
2. Estimate the distributions of public inflation perceptions and expectations in the UK during 2001Q1–2015Q4
3. Illustrate a possible use of the estimated distributions by measuring information rigidity in the UK

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Plan

BoE/GfK NOP Inflation Attitudes Survey

- Quarterly survey started in 2001Q1
- Quota sample of adults aged 16 or over in 175 randomly selected areas throughout the UK
- Quota is 4,000 (Q1) or 2,000 (others)
- Use 60 samples (2001Q1–2015Q4) separately

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Perception

Which of the options on this card best describes how prices have changed over the last twelve months?

Expectation

And how much would you expect prices in the shops generally to change over the next twelve months?

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Intervals

8 intervals (excluding 'No idea'):

1. Gone/Go down
2. Not changed/change (indifference limen)
3. Up by 1% or less
4. Up by 1% but less than 2%
5. Up by 2% but less than 3%
6. Up by 3% but less than 4%
7. Up by 4% but less than 5%
8. Up by 5% or more

Sometimes inappropriate $\Longrightarrow$ 18 intervals since 2011Q2

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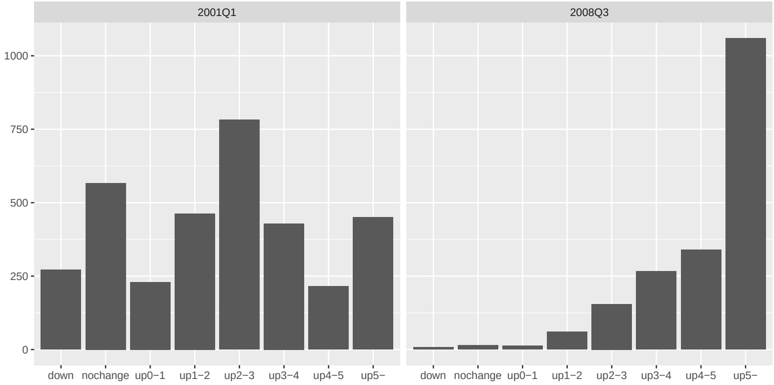
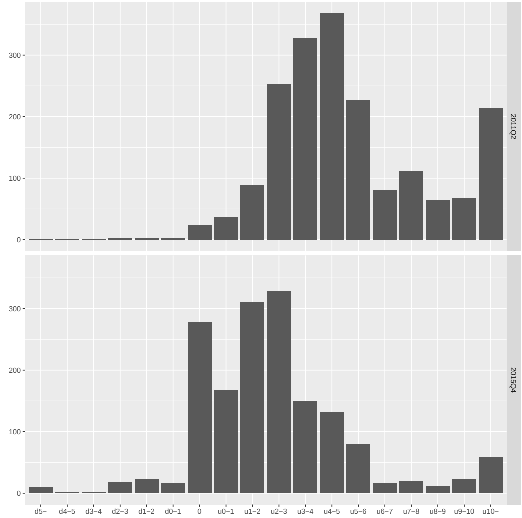
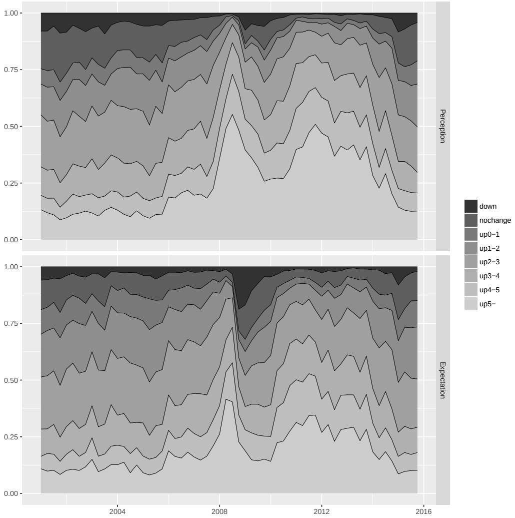
Inflation Perceptions (2001Q1 and 2008Q3)



Inflation Perceptions (2011Q2 and 2015Q4)



Relative Frequencies (2001Q1–2015Q4)



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Interval/Ordinal Response Model

Interval/ordinal response model with an indifference limen

$$y_i := \begin{cases} 1 & \text{if } \gamma_0 < y_i^* \leq \gamma_1 \\ \vdots & \\ J & \text{if } \gamma_{J-1} < y_i^* \leq \gamma_J \end{cases}$$

where

- y_i^* is a latent variable
- $-\infty = \gamma_0 < \cdots < \gamma_l < 0 < \gamma_u < \cdots < \gamma_J = \infty$
- we know $\{\gamma_j\}$ except for an indifference limen $[\gamma_l, \gamma_u]$

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Normal Mixture Model

Normal mixture model for y_i^*

$$y_i^* \sim \sum_{k=1}^{K} \pi_k \mathrm{N}\left(\mu_k, \sigma_k^2\right)$$

Parameters

- $\boldsymbol{\pi} := (\pi_1, \ldots, \pi_K)'$
- $\boldsymbol{\mu} := (\mu_1, \ldots, \mu_K)'$
- $\boldsymbol{\sigma} := (\sigma_1, \ldots, \sigma_K)'$
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Why Bayesian?

ML may not work for normal mixture models

1. Given numerical data, the likelihood function is unbounded with a degenerate component; cf. Kiefer and Wolfowitz (1956)
2. Given interval data, the likelihood function is bounded, but the convergence problem remains; cf. Biernacki (2007)

Bayesian analysis seems safer.

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Prior

Weakly informative priors (i.e., proper but vague) for the distribution parameters

$$\boldsymbol{\pi} \sim \mathrm{Dirichlet}(\boldsymbol{\imath}_K)$$
$$\mu_k \sim \mathrm{N}(2.5, 100), \quad k = 1, \ldots, K$$
$$\sigma_k^2 \sim \mathrm{Inv\text{-}Gam}(2, \beta_0), \quad k = 1, \ldots, K$$

A hierarchical prior for β_0 (Richardson and Green (1997))

$$\beta_0 \sim \mathrm{Gam}(.2, .1)$$

Flat priors for the indifference limen

$$\gamma_l \sim \mathrm{U}(-\infty, 0)$$
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Posterior Simulation

1) Problem with MCMC for cutoff points with large samples

- **Gibbs** Extremely slow
- **MH** Faster (with collapsing and reparametrization), but low acceptance rate (20–50%)
- **NUTS** Much faster with high acceptance rate (80–99%), easy to implement using Stan

2) Label switching problem

- Component labels may switch during MCMC
  $\Longrightarrow$ Mixture parameters may not converge
- Focus on permutation invariant parameters (moments and quantiles); cf. Geweke (2007)

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1) Problem with MCMC for cutoff points with large samples

Gibbs	Extremely slow
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Results Using 6 or 8 Intervals

- Compare estimation using 6 intervals (no indifference limen) and 8 intervals (with an indifference limen)
- Estimation period: 2001Q1–2015Q4
- $K = 2$
- Focus on up to the 5th (central, standardized) moments
  1. mean
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Findings

1. Estimates of the means are reasonably precise with narrow error bands.
2. Estimates of higher-order moments are imprecise.
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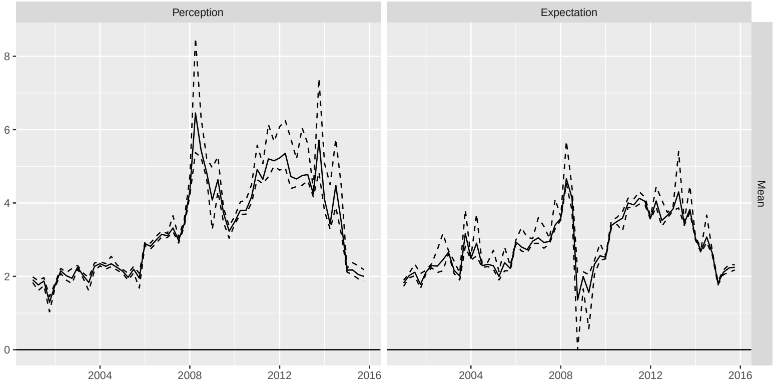
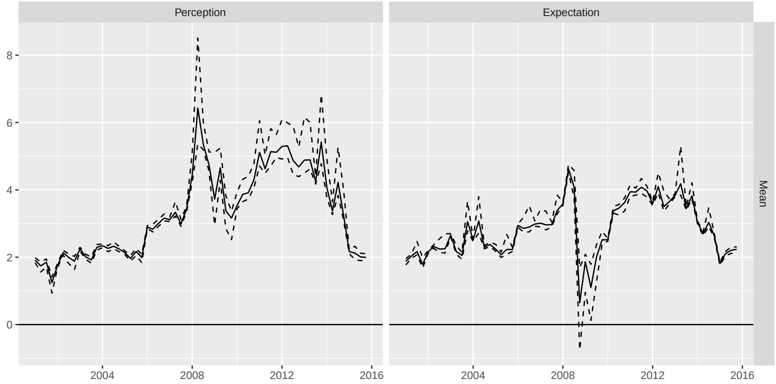


Posterior median and 68% error band

Mean (8 Intervals)



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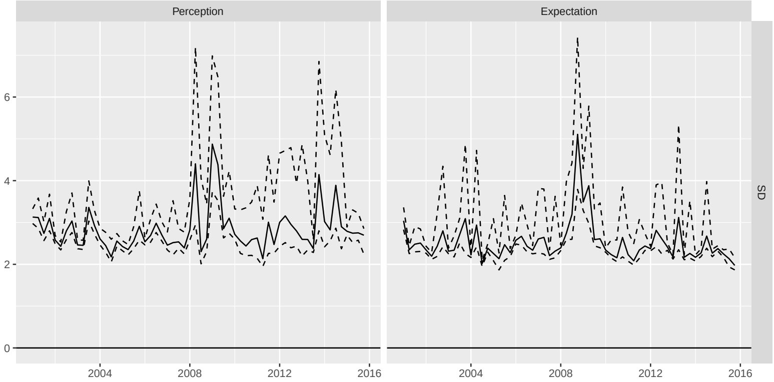

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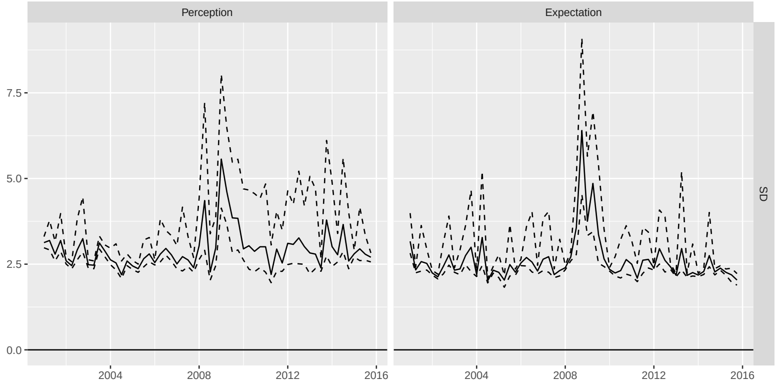

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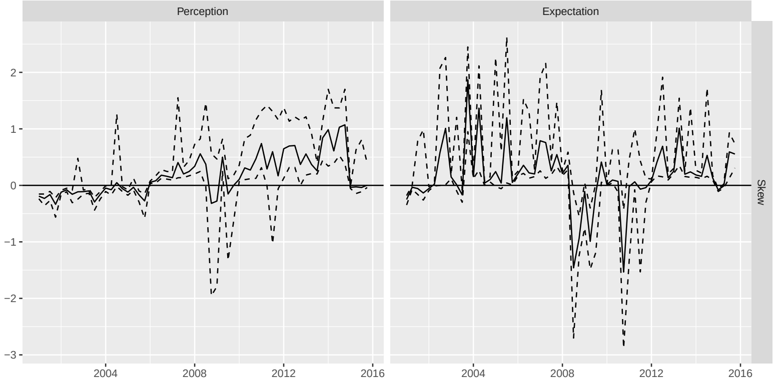

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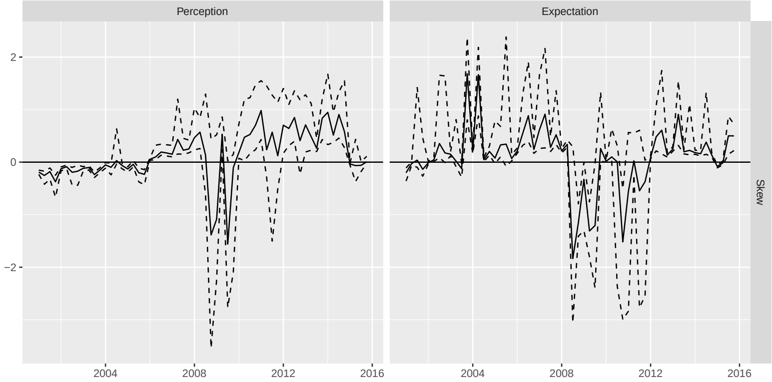

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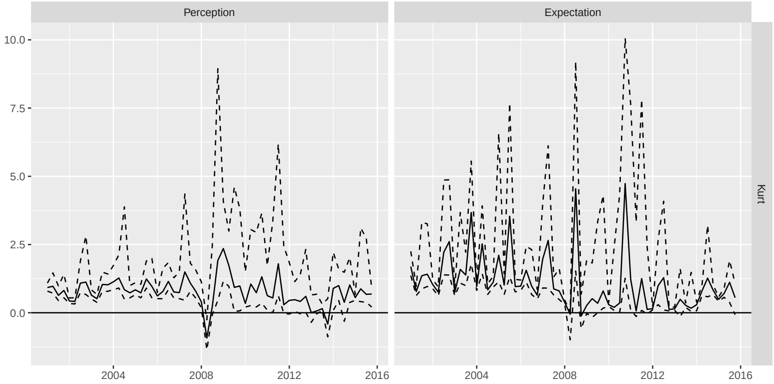
Excess Kurtosis (6 Intervals)



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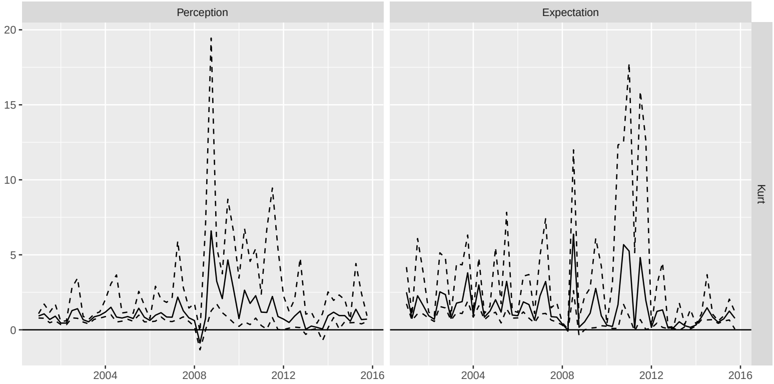

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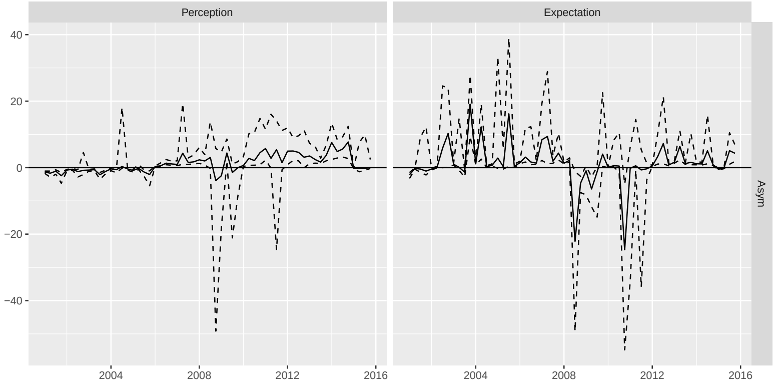
Asymmetry in the Tails (6 Intervals)



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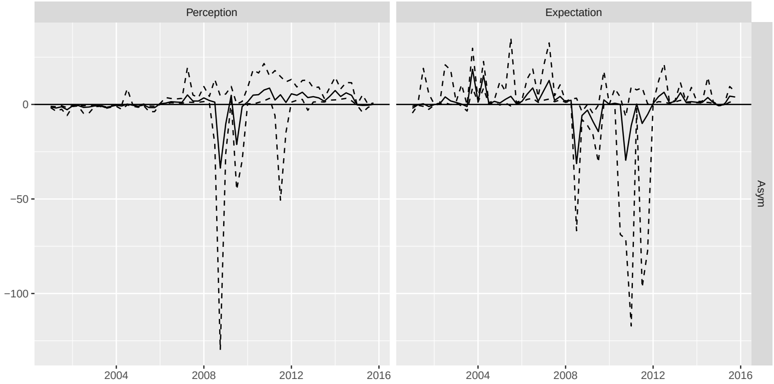

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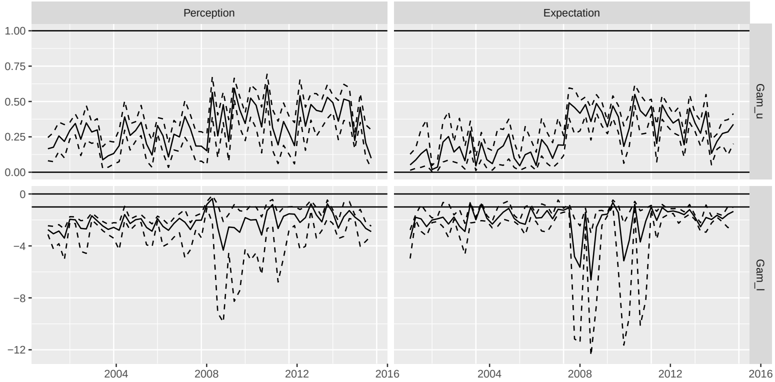

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Results Using 18 Intervals

- Estimation period: 2011Q2–2015Q4
- $K = 2$
- New prior: $\gamma_l \sim \mathrm{U}(-1, 0)$

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Lesson: Design of intervals is crucial, especially when collecting interval data repeatedly.

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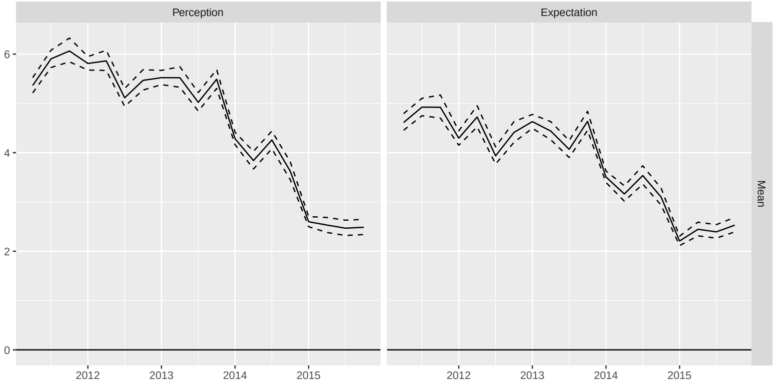
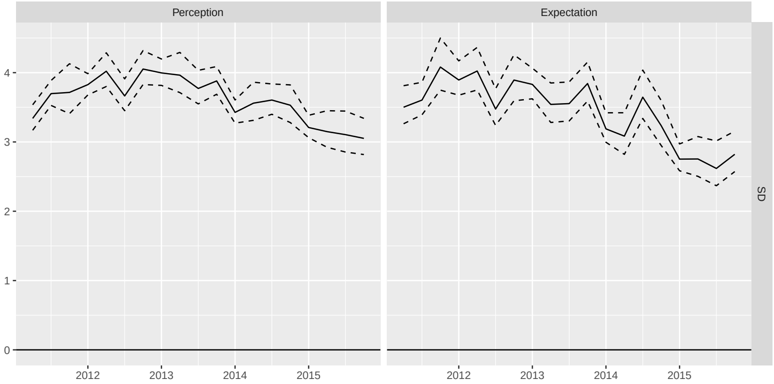


Posterior median and 95% error band

S.D.



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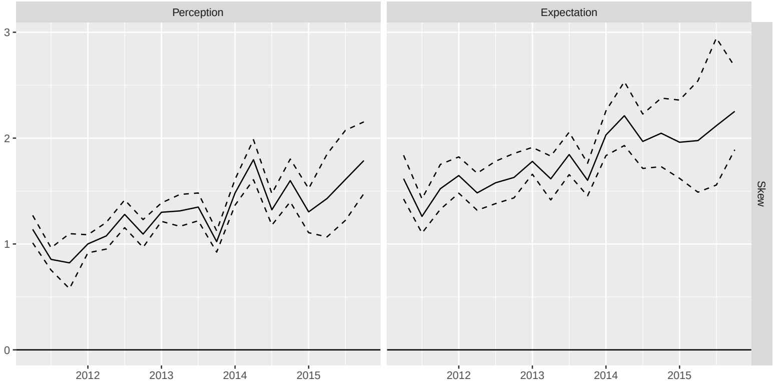
Skewness



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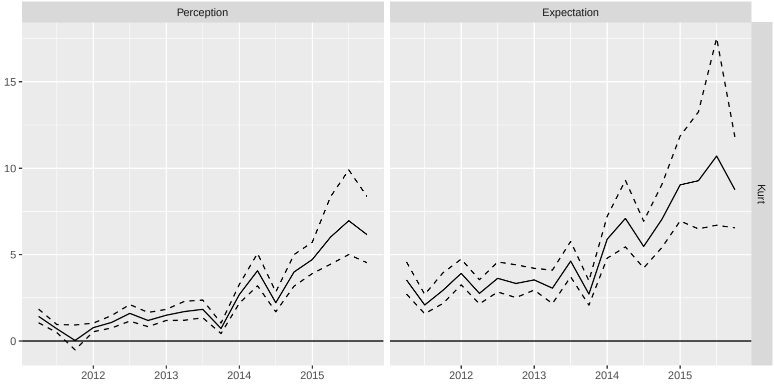

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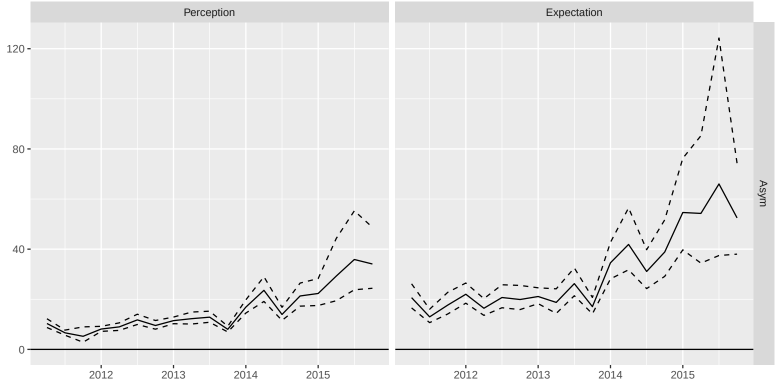

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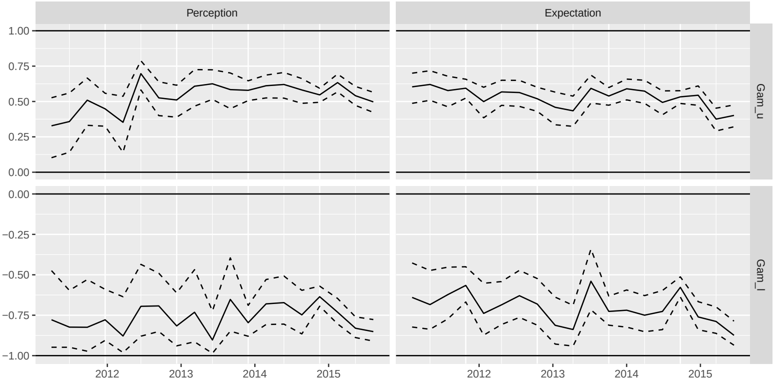

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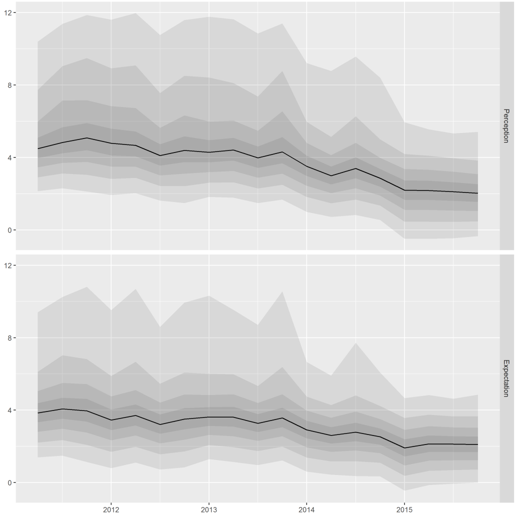

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Mean Inflation Expectations ($K = 1, 2$)

Use of the Estimated Means

Can use the mean inflation expectations for further analyses

- testing for rationality of expectations
- measuring information rigidity; cf. Coibion and Gorodnichenko (2015, AER)

The results depend on how we specify the distribution of inflation expectations $\Longrightarrow$ Use a flexible distribution

(Please see the paper for more details)

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2. Estimate the distributions of public inflation perceptions and expectations in the UK during 2001Q1–2015Q4
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Future Plans

Possible extensions

1. Heterogeneous indifference limens among individuals
2. Estimation of K (number of mixture components)
3. Time series analysis of repeated cross sections using a state space model

Use of individual data (now available!)

1. Determinants of inflation perceptions and expectations; cf. Blanchflower and MacCoille (2009)
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