Bayesian multivariate Beveridge–Nelson decomposition of I(1) and I(2) series with cointegration

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Highlights
- Derive a formula for the multivariate B–N decomposition of I(1) and I(2) series with cointegration
- Develop a Bayesian method to obtain error bands for the components
- Apply the method to US data to obtain a joint estimate of the stochastic trends and gaps of output, inflation, interest, and unemployment

(Multivariate) B–N decomposition
- Decompose I(1) series into
  - random walk permanent component (−LR forecast)
  - I(0) transitory component (−forecastable movement)
- Assuming a linear time series model (ARIMA, VAR, VECM, etc.)
- For non-US data, often “unreasonably persistent” transitory component (perhaps because of structural breaks)
- Extension to I(2) series available

Theoretical motivation
Let
- \( y_t \): log output
- \( r_t \): real interest rate

Consumption Euler equation \( \Rightarrow \) dynamic IS curve:
\[
E_t (\Delta y_{t+1}) = \frac{1}{\sigma} (\gamma - \rho)
\]

Observations:
- \( (r_t) \) is I(1) \( \Rightarrow \) \( \{\Delta y_t\} \) is I(1) \( \Rightarrow \) \( \{y_t\} \) is I(2)
- Cointegration between \( \{\Delta y_t\} \) and \( \{r_t\} \)
- Decompose \( \{y_t\} \) (not \( \{\Delta y_t\} \))

VECM and its state space representation
Notations
- \( \{x_t, \delta_t\} \sim I(d) \)
- \( y_t := (x_t', \Delta \delta_t')' \sim CI(1,1) \)

Steady state VECM
\[
\Phi(L) (\Delta y_t - \mu) = A (\Gamma y_t - \beta - \delta (t - 1)) + u_t
\]
where \( \Phi(L) = A + \Gamma - \Gamma L \)
\( \{u_t\} \sim WN(P^{-1}) \)

State vector
\[
s_t := \begin{bmatrix} \Delta y_{t-1} - \mu \\ \Gamma y_t - \beta - \delta t \end{bmatrix}
\]

Linear state space model
\[
s_t = A s_{t-1} + B \delta_t \\
\Delta y_t = \mu + C s_t \\
\{x_t\} \sim WN(h_t)
\]

where
\[
A := \begin{bmatrix} \Phi_t & \ldots & \Phi_{p-1} \\ \Gamma \Phi_t & \ldots & \Gamma \Phi_{p} - \Gamma A \\ \end{bmatrix} , \quad B := \begin{bmatrix} \Sigma_t^{1/2} \\ \Gamma \Sigma_t^{1/2} \end{bmatrix} , \quad C := \begin{bmatrix} I_h \ O \end{bmatrix}
\]

Multivariate B–N decomposition

B–N transitory component in \( x_{t,y} \)
\[
c_{t,y} = -C_t (\Phi_t - A)^{-1} A s_t
\]
B–N transitory component in \( x_{t,r} \)
\[
c_{t,r} = C_t (\Phi_t - A)^{-1} A^2 s_t
\]

Note: Observable given the parameters \( \Rightarrow \)
- Estimated of \( \{s_t\} \) (by state smoothing) is unnecessary.
- Easy to construct error bands (Gibbs sampler).

Data
Notations
- \( x_t \): CPI inflation rate
- \( r_t \): ex post real interest rate (3-month treasury bill)
- \( u_t \): unemployment rate
- \( Y_t \): real GDP

Let
\[
x_t := \begin{pmatrix} x'_{t,y} \\ x'_{t,r} \end{pmatrix} \quad \quad \quad y_t := \begin{pmatrix} y'_{t,r} \\ u_t \end{pmatrix}
\]

Estimation period: 1950Q1–2017Q4

Bayesian computation
Set \( p := 7 \)
- Use “weakly informative” priors.
- Use ML estimate for initial values of \( (\Phi, P, A, \gamma) \).
- Discard initial 1,000 draws. Use next 4,000 draws for posterior inference.
- From the S–D density ratios (for \( r = 1, 2, 3 \)) vs \( r = 0 \), we find \( r = 2 \) with posterior probability (numerically) 1.
- Use posterior median for point estimate.
- Use posterior 2.5 and 97.5 percentiles for 95% error band.

Interpretation
- I(2) log output \( \Rightarrow \) I(1) output growth rate
- \( \Rightarrow \) Stochastic trend captures structural breaks
- \( \Rightarrow \) “Reasonable” transitory components
- Cointegration \( \Rightarrow \) VECM forecasts better than VAR
- \( \Rightarrow \) “Bigger” transitory components \( (= \) forecastable movements) for all variables (not only for output)